I-PRE BOARD EXAMINATION
MATHEMATICS

(Maximum Marks: 80)
(Time allowed: Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C. Candidates are required to attempt all questions from Section A and all questions
EITHER from Section B OR Section C

Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B: Internal choice has been provided in one question of two marks each and one question of four marks.

Section C: Internal choice has been provided in one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION - A (65 Marks)

Question 1 **[15×1]**

In sub parts (i) to (X) choose the correct options and in sub parts (xi) to (xv), answer the questions as indicated.

(i) Relation R in the set Z of all integers defined as $R=\{(x, y) : |x - y| \text{ is an integer}\}$ is :

(a) reflexive only

(b) symmetric only

(c) transitive only

(d) equivalence relation

This paper consists of 10 printed pages.

(ii) The function $f: W \rightarrow W$ defined as $f(x) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$ is

(a) injective but not surjective

(b) injective and surjective

(c) surjective but not injective

(d) neither surjective nor injective

(iii) If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ then value of $x + y + xy$ is :

(a) 1

(b) 2

(c) - 1

(d) - 2

(iv) Value of determinant $\begin{vmatrix} 1/a & a & bc \\ 1/b & b & ca \\ 1/c & c & ab \end{vmatrix}$ is equal to

(a) 0

(b) a + b + c

(c) abc

(d) a + b – c

(v) If matrix $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ then $A^{-1} =$

(a) $\frac{1}{2}(A - 9I)$

(b) A - 9I

(c) $\frac{1}{2}(9I - A)$

(d) 9I - A

(vi) The value of $\sin^{-1}\left[\cos\left(\frac{43\pi}{5}\right)\right]$ is :

(a) $\frac{\pi}{10}$

(b) $-\frac{\pi}{10}$

(c) $\frac{3\pi}{5}$

(d) None of these

(vii) A be a square matrix of order 3×3, then $|kA|$ is equal to :

(a) $k|A|$

(b) $k^2|A|$

(c) $k^3|A|$

(d) $3k|A|$

(viii) Value of $\int \frac{\cot(\log x)}{x} dx$ is :

(a) $\log|\sin(\log x)| + c$

(b) $\sin|\log(\log x)| + c$

(c) $\log|\log(\sin x)| + c$

(d) none of these

(ix) Degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = 5.\frac{d^2y}{dx^2}$ is :

(a) 1

(b) 2

(c) 3

(d) not defined

(x) If $P(A) = 0.8$, $P(A \mid B) = 0.8$ and $P(A \cap B) = 0.5$ then $P(A \cup B)$ is :

(a) $\frac{7}{40}$

(b) $\frac{17}{40}$

(c) $\frac{27}{40}$

(d) $\frac{37}{40}$

(xi) Events A and B are such that $P(A \mid B) = 0.4$, $P(B \mid A) = 0.25$ and $P(A \cup B) = 0.12$. Are the events independent ?

(xii) If two dice are thrown simultaneously find the probability of getting a sum of 7 or 11.

(xiii) An edge of a variable cube is increasing at a rate of 3 cm/sec. How fast is the volume of cube increasing, when the edge is 10 cm long ?

(xiv) Find the turning points of the function $f(x) = -x^3 + 12x^2 - 5$

(xv) If the matrix $\begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find the value (s) of x.

Question 2 [2]

Prove that the greatest integer function $f: R \rightarrow R$, given by $f(x)=[x]$ is neither one-one nor onto.

Question 3 [2]

Find the intervals in which the function $f(x) = \sin x + \cos x,\ 0 \leq x \leq 2\pi$ is strictly increasing.

Question 4 [2]

(a) Prove that curves $x = y^2$ and $xy = k$ cut orthogonally if $8k^2 = 1$

OR

(b) Find the equation of normal to curve $x^2 = 4y$ which passes through the point (1, 2).

Question 5 [2]

(a) For what choice of a and b is the function $f(x) = \begin{cases} x^2 & x \leq c \\ ax+b & x > c \end{cases}$ differentiable at $x = c$.

OR

(b) If $\sin y = x \sin (a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Question 6 [2]

Solve the differential equation $(1 + y^2)(1 + \log x)\, dx + x dy = 0$

Question 7 [4]

Prove that : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2} + \cos^{-1} x,\ -\frac{1}{2} \leq x \leq 1$

Question 8 [4]

If $y = (\sin^{-1}x)^2$ Prove that $(1 - x^2)\, y_2 - xy_1 = 2$

Question 9 [4]

(a) Evaluate : $\int (\cos x + 3\sin x)e^{3x}\,dx$

OR

(b) Evaluate : $\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}dx$

Question 10 [4]

(a) An urn contains 10 while and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from first urn and put into the second urn. Then a ball is drawn from second urn. Find the probability that the ball drawn from second urn is white.

OR

(b) A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game then find the probability that B wins.

Question 11 [6]

Solve the following system of linear equations using Martin's Rule

$3x + y + z = 1$

$2x + 2z = 0$

$5x + 2z + y = 2$

Question 12 [6]

(a) Solve the differential equation

$\frac{dy}{dx} - 3y\cot x = \sin 2x$ given that $y = 2$ when $x = \frac{\pi}{2}$

OR

(b) Evaluate : $\int \frac{dx}{9\cos^2 x + 8\sin^2 x + 3}$

Question 13 [6]

(a) Show that height of a cylinder of greatest volume which can be inscribed in a right circular cone, is one third that of the cone.

OR

(b) A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of retangle so that it may produce the largest area of the window.

Question 14 [6]

(a) Assume that on an average one telephone number out of fifteen called between 2 p.m. and 3 p.m. on week days are busy, what is the probability that of six randomly selected telephone numbers are called, at least three of them will be busy ?

SECTION - B (15 Marks)

Question 15 [5×1]

In sub–parts (i) and (ii), choose the correct options and in sub–parts (iii) to (v), answer as indicated :

(i) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, then the value of t such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$ is :

(a) 0 (b) 5

(c) 4 (d) 2

(ii) The angle between the planes $2x-y+z = 6$ and $x+y+2z = 7$ is :

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(iii) Find the angle between two vector $\vec{a}$ and $\vec{b}$ if $|\vec{a}\times\vec{b}| = \vec{a}\cdot\vec{b}$

(iv) Find a vector of 9 units perpendicular to plane of the vectors $\vec{a} = 2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$

(v) Find the vector equation of plane through (1, 2, 3) and perpendicular to lime $\frac{x}{-2} = \frac{y}{4} = \frac{z}{3}$

Question 16 [2]

(a) If $\vec{p}$ and $\vec{q}$ are unit vectors forming an angle of 30°, find the area of parallelogram having $\vec{a} = \vec{p}+2\vec{q}$ and $\vec{b} = 2\vec{p}+\vec{q}$ as it's diagonals.

OR

(b) Find the vector projection of $\vec{b}$ on $\vec{a}$ when $\vec{a} = \hat{i}-2\hat{j}+\hat{k}$ and $\vec{b} = 7\hat{i}-3\hat{k}+\hat{j}$

Question 17 [4]

(a) Find the equation of plane passing through points (–1, 1, 1), and (1, –1, 1) and perpendicular to plane $x + 2y + 2z = 5$

OR

(b) If the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{1-x}{-2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect each other, find their point of intersection.

Question 18 [4]

Find the area bounded by the curve $y = x^2$ and $y = 2x-x^2$

SECTION - C [15 Marks]

Question 19 [5×1]

In sub–parts (i) and (ii) choose the correct options and in sub–parts (iii) to (v), answer as instructed.

(i) If R(x) and C(x) represent revenue and total cost functions respectively, then at break even point :

(a) $R(x) < C(x)$ (b) $R(x) = C(x)$

(c) $R(x) > C(x)$ (d) $R(x) = \frac{1}{2}C(x)$

(ii) The two regression lines intersect at the point (4, 5) then mean for variate x is :

(a) 4 (b) 5

(c) 4.5 (d) 9

(iii) If total cost function for a manufacture is given by

$$C(x) = \frac{5x^2}{\sqrt{x^2+3}} + 50C$$ then find average cost function.

(iv) The demand function for a certain product is $p = 20 + 5x - 3x^2$, where x is the number of units demanded and p is the price per unit find marginal revenue function.

(v) The regression coefficient of x on y is 0.8 and correlation coefficient between x and y is 0.4 then find the value of regression coefficient of y on x.

Question 20 [2]

(a) If $C(x) = \sqrt{6x+5} + 2500$, show that marginal cost decreases as output x increases.

OR

(b) For demand function $p = \frac{b}{a+x}$, show that marginal revenue function is increasing for all $b < 0$ and $a > 0$.

Question 21 [4]

(a) Given the data

x	1	5	3	2	1	1	7	3
y	6	1	0	0	1	2	1	5

find regression line of x on y and hence predict x wen y = 2.5 .

OR

(b) If the two regression lines are $4x - 2y = 3$ and $2x - 3y = 5$, find correlation coefficient between x and y. Also predict the value of y when value of x is 3.

Question 22 [4]

To maintain one's health, a person must fulfil certain minimum daily requirements for the following three nutrients, Calcium, Protein and Calories. His diet consists only food I and food II whose price and nutrient contents are shown below :

Price	Food I ₹0.60/ unit	Food II ₹1.00/ unit	Minimum daily requirement
Calcium	10	4	20
Protein	5	5	20
Calories	2	6	12

Set up the linear programming problem and solve it graphically, the objective function being the minimisation of cost for the combination of food units.

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