I- TERM EXAMINATION : 2021-22

CLASS - XII (ISC)

Time : 3 hrs. **MATHEMATICS PAPER-1 (THEORY)** M.M.: 80

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time.)

The questions paper consists of three sections A, B and C.

Candidates are required to attempt all questions from Section A and all questions**Either** from Section B **OR** Section C.

SECTION A : Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.

SECTION B : Internal choice has been provided in one questions of two marks and one questions of four marks each.

SECTION C : Internal choice has been provided in one questions of two marks and one question of four marks.

All working including rough work should be done on the same sheet is, and adjacent to the rest of the answer. The intended marks for questions or parts of questions are given in bracket [ ]. Mathematical tables and graph papers can be used.

SECTION - A [65 Marks]

Q.1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed : [15×1=15]

(i) If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then adj A= ?

(a) $\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$ (b) $\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$ (c) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (d) $\begin{bmatrix} -d & -b \\ c & a \end{bmatrix}$

(ii) Range of $\text{cosec}^{-1}x$ is :

(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ (d) none of these

(iii) If A is a 3-rowed square matrix and $|3A|=k|A|$ then k=?

(a) 3 (b) 9 (c) 27 (d) 1

(iv) $f:R\to R$: f(x) cosx is :

(a) one-one and into (b) one-one and onto

(c) many-one and into (d) many-one and onto

(v) Let $f:Q\to Q:f(x)=(2x+3)$, then $f^{-1}(y)$=?

(a) 2y - 3 (b) $\frac{1}{2y-3}$ (c) $\frac{1}{2}(y-3)$ (d) none of these

(vi) Let R be a relation on the set N of all Natural numbers, defined by ${}_aR_b$ s.t. a is greater than b. Then R is :

(a) only reflexive (b) only symmetric

(c) only transitive (d) none of these

(vii) $\tan^{-1}(-1)+\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ =?

(a) $\pi/2$ (b) π (c) 0 (d) $2\pi/3$

(viii) For A 2×2 order matrix and if $A\cdot(\text{adj}A)=\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ then the value of $|A|$ is :

(a) 0 (b) 8 (c) 64 (d) 4

(ix) $f(x) = x^{1/3}$ is :

(a) continuous at x=0 (b) differentiable at x=0

(c) continuous and differentiable both at x=0

(d) none of these

(x) If the points (3, -2), (k, 2) and (8, 8) are collinear then the value of k is :

(a) 2 (b) - 3 (c) 5 (d) - 4

(xi) Find the value of k for which $f(x) = \begin{cases} \frac{\sin 2x}{5x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x = 0

(xii) Differentiate $e^{\sqrt{x}}$ w.r.t. 'x'.

(xiii) Find $\frac{dy}{dx}$ if $x^3 + y^3 = a^3$

(xiv) Solve for x : $\cos\left(\sin^{-1} x\right) = \frac{1}{2}$

(xv) Find (AB)' if $A = \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 6 & -4 \end{bmatrix}$

Q.2. Verify Rolle's Theorem for $f(x) = \frac{\sin x}{e^x}$; $x \in [0, \pi]$ [2]

Q.3. Using elementary transformations find the inverse of the matrix $\begin{bmatrix} 9 & 5 \\ 7 & 4 \end{bmatrix}$ [2]

Q.4. Test the continuity of $f(x) = \begin{cases} (x-a)\sin\frac{1}{(x-a)} & ; x \neq a \\ 0 & ; x = 0 \end{cases}$ at x=a . [2]

Q.5. Find the identity for the binary operation*defined as a* b = a+b+1 ; a, b $\in$ Z [2]

OR

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto

Q.6. Prove that : $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$ [2]

OR

Evaluate : $\sin\left[2\tan^{-1}\frac{3}{5} - \sin^{-1}\frac{7}{25}\right]$ [2]

Q.7. Verify LMVT for the function f(x) = 2sinx+sin2x on [0, π] [4]

Q.8. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2+4x-42=0$. Hence find A^{-1} . [4]

OR

Using properties of determinants, prove that $\begin{vmatrix} y+z & x+y & x \\ z+x & y+z & y \\ x+y & z+x & z \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz$

Q.9. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{2-\log x}{(1-\log x)^2}$ [4]

Q.10. Show that $\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1+2\cos x}{2+\cos x}\right)$ [4]

OR

Evaluate : $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$ where a>b>c>0

Q.11. If $A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}$, Find A^{-1} and hence solve the system of equations 4x-5y-11z=12 ; x-3y+z=1, 2x+3y-7z=2 [6]

OR

Find the product of $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & -6 \\ 3 & -2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 20 & 2 & 34 \\ 8 & 16 & -32 \\ 22 & -13 & 7 \end{bmatrix}$ and use it to solve the system of equations given below :

$\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = -3$; $\frac{5}{x} + \frac{4}{y} - \frac{6}{z} = 4$; $\frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 6$

Q.12. If $y = \log\left\{x + \sqrt{x^2 + a^2}\right\}$, then prove that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ [6]

Q.13. Using properties of determinants show that $p\alpha^2 + 2q\alpha + r = 0$, given that p, q, r are not in G.P. and $\begin{vmatrix} 1 & q/p & \alpha + q/p \\ 1 & r/q & \alpha + r/q \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$ [6]

Q.14. Let I be the set of all integers and R be the relation on I defined by $_aR_b$ iff (a+b) is an even integer $\forall a, b \in I$. Prove that R is an equivalence relation. [6]

OR

Let A=R×R and * be the binary operation on A defined by (a, b) * (c, d)=(a+c, b+d). Show that * is commutative and associative. Find the identify element for * on A, if any.

SECTION - B [15 Marks]

Q.15. In subparts (i) , (ii) and (iii), choose the correct options and in subparts (iv) and (v) answer the questions as instructed : [1×5=5]

(i) If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $(3\vec{a} + 2\vec{b})(5\vec{a} - 6\vec{b}) = ?$

(a) 3 (b) 5 (c) - 9 (d) - 1

(ii) If $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \cdot \vec{b}| = 35$ then $\vec{a} \cdot \vec{b} = ?$

(a) 5 (b) 7 (c) 13 (d) 12

(iii) $[\hat{i} \quad \hat{j} \quad \hat{k}] = ?$

(a) 0 (b) 1 (c) 2 (d) 3

(iv) If $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$ find $|\vec{a} \cdot \vec{b}|$

(v) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$. Evaluate $[\vec{a} \quad 2\vec{b} \quad \vec{c}]$

Q.16. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \cdot \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$ [4]

OR

Find the sine of the angle between the vectors $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

Q.17. Show that $\vec{a}, \vec{b}, \vec{c}$ are non coplanar iff $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ are non coplanar. [2]

OR

If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$

Q.18. Find by vector method , the area of triangle ABC whose vertices are A(1, 2, 3), B(2, 5, -1) and C(-1, 1, 2). [4]

SECTION - C [15 Marks]

Q.19. In subparts (i) to (iii) choose the corect options and in subparts (iv) and (v), answer the questions as instructed : [1×5=5]

(i) If line of best fit is 8x-10y+66=0, estimate y when x=2.

(a) 4.2 (b) 8.2 (c) 6.7 (d) None of these

(ii) Given that standard deviations of X and Y are 7 and 4 respectively and the correlation coefficient is 0.86 . The regression coefficient is :

(a) 0.491 (b) 1.505 (c) 3.44 (d) 0.571

(iii) The regression coefficient b_{yx} is :

(a) $r\frac{\sigma x}{\sigma y}$ (b) $\frac{r\sigma y}{\sigma x}$ (c) $\frac{\sigma y}{r\sigma x}$ (d) $\frac{\sigma x}{\sigma y}$

(iv) If $b_{yx} = 0.89$ and $b_{xy} = 0.85$, then find the coefficient of correlation.

(v) Out of the two regression lines, find the line of regression Y on X.

3x + 12y = 8, 9x +3y = 46

Q.20. The lines of regression in a bivariate distribution are x+2y=6, 2x+3y-8=0 Find the mean of x and y. [2]

OR

If the two regression lines of a bivariate distribution are 4x-5y+33=0 and 20x-9y-107=0, find the variance of y when σx=3

Q.21. Find the line of best fit using x as dependent variable. [4]

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

OR

For observation of pairs (x, y) of the variables X and Y, the following results are obtained $\sum x = 125$, $\sum y = 100$, $\sum x^2 = 1650$, $\sum y^2 = 1500$, $\sum xy = 50$, n=25. Find the equation of the line of regression of x on y. Estimate the value of x if y=5 .

Q.22. To find regression coefficients following data were arranged. [4]

	x	y
Arithmetic mean	36	85
Standard deviation	11	8

Correlation coefficient between x and y=0.66. Find the two regression equation and most likely value of y when x=10.

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