I -TERM EXAMINATION : 2023-24

CLASS - XII (CBSE)

MATHEMATICS

Time: 3 hrs. M.M.: 80

General Instructions:

1) *This question paper contains- five* **Sections** *A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2) **Section–A** *has 18* **MCQ's and** *02 Assertion-Reason based questions of 1 mark each.*
3) **Section–B** *has 5* **Very Short Answer (VSA)**-*Type questions of 2 marks each.*
4) **Section–C** *has 6* **Short Answer (SA)**-*Type questions of 3 marks each.*
5) **Section–D** *has 4* **Long Answer (LA)** -*Type questions of 5 marks each.*
6) **Section–E** *has 3 source* **based/case based/passage based/integrated units of assessment** *(4 marks each) with sub parts.*

SECTION - A

(MULTIPLE CHOICE QUESTIONS)

(Each question carries 1 mark)

Q.1. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2 ; b > 6\}$. Choose the correct answer.

a) $(2,4) \in R$ b) $(3,8) \in R$

c) $(6,8) \in R$ d) $(8,7) \in R$

Q.2 Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$ then number of one-one functions from A to B are :

a) 1 b) 2 c) 12 d) 0

Q.3. Let T be set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$ then R is :

a) an equivalence relation

b) not symmetric relation

c) not reflexive relation

d) not transitive relation

Q.4. $\cos^{-1}\cos\left(\frac{-17\pi}{4}\right)$ is equal to :

a) $\frac{\pi}{4}$ b) $\frac{17\pi}{4}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{4}$

Q.5. $\cos^{-1} x$ is equal to :

a) $2\sin^{-1}\sqrt{\frac{1+x}{2}}$ b) $2\cos^{-1}\sqrt{\frac{1-x}{2}}$

c) $2\cos^{-1}\sqrt{\frac{1+x}{2}}$ d) none of these

Q.6. The principal value of the expression $\cos^{-1}[\cos(-680^\circ)]$ is :

a) $\frac{2\pi}{9}$ b) $\frac{-2\pi}{9}$ c) $\frac{34\pi}{9}$ d) $\frac{\pi}{9}$

Q.7. If A and B are symmetric matrices, then AB – BA is a :

a) symmetric matrix

b) skew symmetric matrix

c) diagonal matrix

d) unit matrix

Q.8. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then $A^2-5A-7I$ is :

a) a zero matrix
b) identity matrix
c) diagonal matrix
d) none of these

Q.9. If A is square matrix such that $A^2=A$ then $(I+A)^5-8A$ is equal to :

a) A
b) 18 I
c) 24 I
d) none of these

Q.10. If $\begin{vmatrix}x & 2\\ 18 & x\end{vmatrix}=\begin{vmatrix}6 & 2\\ 18 & 6\end{vmatrix}$, then x is equal to :

a) 6
b) ±6
c) - 6
d) 0

Q.11. If area of triangle is 35 sq. units with vertices (2, - 6), (5, 4) and (k, 4). Then k is :

a) 12
b) -2
c) -12, -2
d) 12, -2

Q.12. If $A=\begin{bmatrix}2 & 3 & -3\\ 0 & 2 & 5\\ 1 & 1 & \lambda\end{bmatrix}$ then A^{-1} exists if

a) $\lambda \neq \frac{-11}{4}$
b) $\lambda = -\frac{11}{4}$
c) any value of λ
d) none of these

Q.13. If $x=at^2, y=2at$ then $\frac{d^2y}{dx^2}$ is :

a) $\frac{1}{t}$
b) $-\frac{1}{t^2}$
c) $a\,t^2$
d) $-\frac{1}{2at^3}$

Q.14. Derivative of cot x° with respect to x is :

a) cosec x°
b) cosecx° cot x°
c) $-1^\circ \operatorname{cosec}^2 x^\circ$
d) $-1^\circ \operatorname{cosec} x^\circ \cot x^\circ$

Q.15. The function f(x)= | sinx | is :

a) continuous for all real number of x.
b) discontinuous for all real number of x.
c) continuous when x = 0
d) none of these

Q.16. The maximum value of $\left(\frac{1}{x}\right)^x$ is :

a) e
b) e^e
c) $e^{1/e}$
d) $\left(\frac{1}{e}\right)^{1/e}$

Q.17. The interval on which the function $f(x)=x^2 \cdot e^{-x}$

a) (0, 2)
b) $(2,\infty)$
c) $(-\infty,0)$
d) $(-\infty,0)\cup(2,\infty)$

Q.18. If $V = \frac{4}{3}\pi r^3$, at what rate in cubic units is V increasing when r = 10 cm and $\frac{dr}{dt} = 0.01$

a) π b) 4π c) 40π d) $4\pi/3$

Assertion-Reason Based Question

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true

Q.19. **Assertion (A) :** If k is a scaler and A is an n×n square matrix. Then $|kA|$ is equal to $k^n|A|$

Reason (R) : If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is $125\,\Delta$.

Q.20. **Assertion (A):** The domain of the function $\sec^{-1}(2x)$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

Reason (R) : $\sec^{-1}(-2) = -\frac{\pi}{4}$

SECTION-B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

Q.21. If A = diagonal [1, –2, 5] and B = diagonal [3, 0, –4] then find 3A – 2B.

Q.22. If $y = \cos\tan\sqrt{x+1}$ find $\frac{dy}{dx}$.

OR

If $y = x^{x^x}$ find $\frac{dy}{dx}$

Q.23. A ladder 13 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/sec. How fast is its height on the wall decreasing when the foot of the ladder if 5 m away from the wall.

Q.24. Prove that $f(x) = \frac{2}{x} + 5$, is a strictly decreasing function.

Q.25. Check injectivity and surjectivity of the function $f: Z \to Z$ defined by $f(x) = x^3$.

OR

Find the domain of the function $f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}$

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

Q.26. Solve the equation : $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2},\ x \neq 0$

Q.27. If the matrix $\begin{bmatrix} -2 & x-y & -2 \\ 1 & 0 & 3 \\ x+y & z & -1 \end{bmatrix}$ is given to be symmetric, find the values of x, y and z.

OR

If $A=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$, check whether that $A-A^T$ is a skew symmetric matrix or not.

Q.28. Find the value of k so that the function f defined by $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x} & \text{if } x\neq\frac{\pi}{2}\\ 3 & \text{if } x=\frac{\pi}{2}\end{cases}$

is continuous at $x=\frac{\pi}{2}$.

Q.29. Differentiate $\cos x \cdot \cos 2x \cdot \cos 3x$ with respect to 3x

OR

Differentiate $e^{ax}\sin^3 x\,\ell og(2x+3)$ w : r. t. 'x'

Q.30. A particle moves along the curve $y=\frac{2}{3}x^3+1$. Find the points on the curve at which the y coordinate is changing twice as fast as the x-coordinate.

Q.31. Prove that $y=\frac{4\sin\theta}{2+\cos\theta}-\theta$ is an increasing function of θ an $\left[0,\frac{\pi}{2}\right]$.

OR

Determine the intervals in which the function $f(x)=3x^4-4x^3-12x^2+5$ is strictly increasing or strictly decreasing.

SECTION -D

(This section comprises of long answer type questions (LA) of 5 marks each.)

Q.32. If $A=\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}$ find A. B. Hence solve the system of equations :

$x-y=3,\quad 2x+3y+4z=17,\quad y+2z=7$

OR

If $A=\begin{bmatrix}1 & 2 & 0\\-2 & -1 & -2\\0 & -1 & 1\end{bmatrix}$, find A^{-1}, using A^{-1}, solve the system of linear equations :

$x-2y=10,\ 2x-y-z=8, -2y+z=7$

Q.33. If $x=\tan\left(\frac{1}{a}\ell og\, y\right)$, show that $(1+x^2)y''+(2x-a)y'=0$

OR

If $y=(\sec^{-1}x)^2, x>1$ show that $x^2(x^2-1)y_2+(2x^3-x)y_1-2=0$.

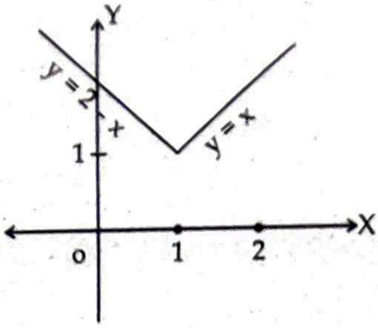
Q.34. Let N be the set of all natural numbers and R be the relation on N × N defined by (a . b), R(c.d) iff $ad(b+c)=bc(a+d)$ for all a, b, c, d ∈ N. Show that R is an equivalence relation.

Q.35. Show that a right circular cylinder which is open at the top, and has given surface area will have the greatest volume if its height is equal to the radius of the base.

SECTION - E

This section comprises of 3 case study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study questions has two sub parts of 2 marks each.

Q.36. The graph of a function $f : R \rightarrow R$ defined by $f(x) = |x-1| + 1$ is given below :



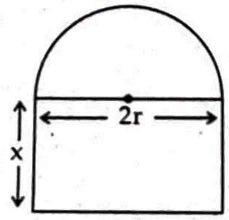
Based on the above information, answer the following questions :

i) Find the range of the function

ii) Determine the function f is whether injective or not

iii) What is the maximum value of the function f ?

Q.37. An interior decorator designs the interiors of a new building. He designs the wall board in the form of a rectangle surmounted by a semicircular design. He is asked to keep the perimeter of the doors 30 m, as shown.



Based on the above information answer the following :

i. Write relation between x and r with respect to perimeter.

ii. Write area of the whole wall board.

iii. Find area of the wall board in terms of r.

OR

Write the value of r for which area is maximum.

Q.38. A diet is to contain 30 units of vitamin A, 40 units of vitamin B and 20 units of vitamin C. Three types of foods F_1, F_2 and F_3 are available. One unit of food F_1 contains 3 units of vitamin A, 2, units of vitamin B and 1 unit of vitamin C. One unit of food F_2 contains 1 unit of vitamin A, 2 unit of vitamin B and 1 unit of vitamin C. One unit of food F_3 contains 5 units of vitamin A, 3 units of vitamin B and 2 units of vitamin C.

If the diet contains x units of food F_1, y units of food F_2 and z units of food F_3.

Based on the above informations answer the following questions :

i) What is the matrix equation representing the above situation ?

ii) If P is the coefficient matrix in above situation then what is the value of $|\text{adj } P|$?

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