# II-PRE BOARD EXAMINATION
# CLASS : XII (ISC)
# MATHEMATICS

*(Maximum Marks: 80)*
*(Time allowed: Three hours)*

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*(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time.)*

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*The Question Paper consists of three sections A, B and C. Candidates are required to attempt all questions from Section A and all questions*
***EITHER** from Section B **OR** Section C*

*Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*

*Section B: Internal choice has been provided in one question of two marks each and one question of four marks.*

*Section C: Internal choice has been provided in one question of two marks and one question of four marks.*

*All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*

*The intended marks for questions or parts of questions are given in brackets [ ].*

*Mathematical tables and graph papers are provided.*

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## SECTION - A (65 Marks)

*Question 1* **[15×1]**

In sub parts (i) to (x) choose the correct options and in sub parts (xi) to (xv), answer the questions as indicated.

(i) Let R be the relation in the set {1, 2, 3, 4} given by R={(1, 2), (2, 2), (1, 1) , (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(a) R is reflexive and symmetric but not transitive

(b) R is reflexive and transitive but not symmetric

(c) R is symmetric and transitive but not reflexive

(d) R is an equivalence relation

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**This paper consists of 10 printed pages.**

(ii) Let $f(x) = 5x - 3$, $f : R \to R$. Choose the correct answer.

(a) f is one - one onto

(b) f is many one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto

(iii) Value of $\sec^2(\tan^{-1} 2) + \mathrm{cosec}^2(\cot^{-1} 3)$ is :

(a) 15 (b) 5

(c) 13 (d) none of the above

(iv) If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then :

(a) $f(a) = 0$ (b) $f(b) = 0$

(c) $f(0) = 0$ (d) $f(1) = 0$

(v) The integrating factor of the differential equation $(x \log x)\frac{dy}{dx} + y = 2\log x$ is :

(a) $e^x$ (b) $\log x$

(c) $\log(\log x)$ (d) $x$

(vi) If A is a matrix of order 3 × 3, then $|3A|$ is

(a) $27\,|A|$ (b) $9\,|A|$

(c) $3\,|A|^3$ (d) none of the above

(vii) The equation of the normal to the curve $y = \sin x$ at (0, 0) is :

(a) $x = 0$ (b) $y = 0$

(c) $x + y = 0$ (d) $x - y = 0$

(viii) The slope of the tangent to the curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point (2, –1) is :

(a) $-7/6$  (b) $6/7$

(c) $7/6$  (d) $-6/7$

(ix) The sum of the degree and the order of the differential equation $y = xy' + \sqrt{1 + y'^2}$ is

(a) 6  (b) 5

(c) 4  (d) none of the above

(x) If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to :

(a) 1/10  (b) 1/8

(c) 7/8  (d) 17/20

(xi) Find the maximum value of the function $f(x) = \sin x + \cos x$.

(xii) Two cards are drawn simultaneously at random from a pack of 52 cards. What is the probability that the drawn cards are of different suits ?

(xiii) Evaluate :

$$\int_2^8 |x - 5|\, dx$$

(xiv) If $f(x) = 5x$, $f : N \rightarrow N$, then state whether f is bijective or not.

(xv) Find the least value of 'a' such that $f(x) = x^2 - ax + 1$ is decreasing in the interval [1, 2] .

**Question 2** [2]

(a) If $\sin^2 x + \cos^2 y = 1$, find $\frac{dy}{dx}$

OR

(b) The edge of a variable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5 cm long.

**Question 3** [2]

Let A = {1, 2, 3, 4} and B = {5, 6, 7, 8}.

Let $f : A \rightarrow B$ be defined by f : {(1, 5), (2, 6), (3, 6), (4, 7)}.

Show that f is neither one-one nor onto.

**Question 4** [2]

The equation of tangent at (2, 3) on the curve $y^2 = px^3 + q$ is $y = 4x - 7$. Find the values of p and q.

**Question 5** [2]

(a) Evaluate :

$$\int \frac{\sin(x-a)}{\sin x} dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \sin^2 x \cos^3 x \, dx$$

**Question 6** [2]

Solve the differential equation

$$ydx - (x + 2y^2)\,dx = 0$$

**Question 7** [4]

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

**Question 8** [4]

If $y = (\sin^{-1}x)^2$ ; Prove that $(1-x^2)y'' - xy' = 2$

**Question 9** [4]

(a) Prove that :

$$\int_0^{2\pi} \frac{x\cos x}{1+\cos x}\,dx = 2\pi^2$$

OR

(b) Evaluate :

$$\int \frac{1}{3x^4 - 2x^2 + 1}\,dx$$

**Question 10** [4]

(a) Find the mean number of heads in three tosses of a coin.

OR

(b) A and B throw a pair of dice alternately till one of them gets a total of 10 and wins the game. Find their respective probability of winning, if A starts first.

**Question 11** [6]

If $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$, find $A^{-1}$

Using $A^{-1}$, solve the following system of equations.

$x + y - z = 1$

$3x + y - 2z = 3$

$x - y - z = -1$

**Question 12** [6]

(a) Solve the differential equation

$(y + \ell og\ x)\ dx - x\ dy = 0$, given that $y = 0$, $x = 1$

OR

(b) Evaluate :

$$\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\, dx$$

**Question 13** [6]

(a) Find the volume of the largest cone that can be inscribed in a sphere of radius R.

OR

(b) Prove that the perimeter of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.

**Question 14** [6]

Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6 she tosses a coin once and notes whether a 'head or tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ?

## SECTION - B (15 Marks)

***Question 15*** [5×1]

In sub–parts (i) and (ii), choose the correct options and in sub–parts (iii) to (v), answer as indicated :

(i) The value of x for which $x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector :

(a) $\pm\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $-\frac{1}{\sqrt{3}}$ (d) none of these

(ii) If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors. Then the value of $|2\hat{a}+\hat{b}+\hat{c}|$ is :

(a) $2\sqrt{6}$ (b) $\sqrt{6}$

(c) 6 (d) $\pm\sqrt{6}$

(iii) Find a vector of magnitude $\sqrt{171}$, which is perpendicular to both of the vectors $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$.

(iv) A plane passes through points (2, 0, 0), (0, 3, 0) and (0, 0, 4), find the equation of the plane.

(v) Write down a unit vector, making an angle of $30^{\circ}$ with the positive direction of x-axis.

***Question 16*** [2]

(a) If and are two vectors such that $|\vec{a}|=4$, $|\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=6$, find angle between $\vec{a}$ and $\vec{b}$

OR

(b) If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot 3\vec{b} = -8$, find $|\vec{a} + 3\vec{b}|$

***Question 17*** [4]

(a) Find the image of the point having position vectors $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane

$$\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$$

OR

(b) Determine the equation of the line passing through the point (1, 2, - 4) and perpendicular to the two lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{8-z}{5}$$

***Question 18*** [4]

Find the area of region enclosed by the parabola $y^2 = 4ax$ and the line

$y = mx$, $a, m > 0$

**SECTION - C [ 15 Marks ]**

***Question 19*** [5×1]

In sub–parts (i) and (ii) choose the correct options and in sub–parts (iii) to (v), answer as instructed.

(i) If $C(x) = \frac{1}{3}x^3 + x^2 - 8x + 5$ then the marginal cost is :

(a) $\frac{1}{2}x^2 + x - 8$

(b) $x^2 + 2x - 8$

(c) $x + 2 - 8/x$

(d) none of the above

(ii) If $b_{yx} = 0.8$ and $b_{xy} = 0.45$, then $r_{xy}$ is :

(a) 0.6 (b) – 0.6

(c) 0.3 (d) – 0.3

(iii) The revenue R from sale of q units of a commodity is given by $R = 20q - 0.5q^2$. How fast does R change with respect to q ?

(iv) If two regression line are $4x - 5y + 33 = 0$ and $20x - 9y = 107$, then find the mean value of x and y.

(v) If demand function of a monopolist is given by $P = 15000 - 2x - x^2$. Find the marginal revenue.

*Question 20* [2]

(a) The fixed cost of a new product is ₹ 30000 and the variable cost per unit is ` 800. If the demand function is $P(x) = 4500 - 100x$ :

I) Find the cost function

II) Find the revenue function.

OR

(b) Verify that $\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$ given that $C(x) = a + bx + cx^2$

*Question 21* [4]

Find the regression coefficient of line y on x for the data

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| y | – 4 | – 3 | – 1 | 0 | 1 | 3 | 5 | 8 |

and hence find the estimate value y for x = 13.5

**Question 22** [4]

(a) If a young man rides his motor cycle at 25 km / hr he has to spend ₹ 2 per km on petrol, if he rides at a faster speed of 40 km/hr, the petrol cost increases to ₹ 5 per km. He has ₹ 100 to spend on petrol and wishes to find maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it.

OR

(b) A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. The food X and Y are available at a cost of ₹ 4 and ₹ 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, whereas one unit of food Y contains 100 units of vitamins 2 units of minerals and 40 units of calories . Find what combination of X and Y should be used to have least cost satisfying the requirements ?

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