Roll No. | | | | | | | |

Candidate must write the code number on the title page of the answer book.

- Please check that this question paper contains 10 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the Serial Number of the question before attempting it.
- 15 minute time has been allotted to read this question paper. The students will read the question paper only and will not write any answer on the answer-book during this period.

I-PRE BOARD EXAMINATION

MATHEMATICS

Time Allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

1) *This question paper contains- **five Sections** A, B, C, D and E Each Section is compulsory. However, there are internal choices in some questions.*
2) *Section–A has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.*
3) *Section–B has 5 **Very Short Answer (VSA)**-Type questions of 2 marks each.*
4) *Section–C has 6 **Short Answer (SA)**-Type questions of 3 marks each.*
5) *Section–D has 4 **Long Answer (LA)** -Type questions of 5 marks each.*
6) *Section–E has 3 **source based / cased based / passage based / integrated units of assessment** (4 marks each) with sub parts.*

SECTION–A

[Multiple Choice Questions]

Each question carries 1 mark.

Q.1. If A and B are matrices of same order, then (AB' -BA') is a :

(a) Skew-symmetric matrix

(b) Null matrix

(c) Symmetric matrix

(d) Unit matrix

Q.2. For any two matrices A and B, we have

(a) $AB = BA$

(b) $AB \neq BA$

(c) $AB = 0$

(d) None of these

Q.3. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda\vec{a}|$ is :

(a) [0, 8]

(b) [-12, 8]

(c) [0, 12]

(d) [8, 12]

Q.4. The function $f: R \rightarrow R$ defined by $f(x) = 4 - 3\cos x$ is :

(a) bijective

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

Q.5. If $y=\sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{\cos x}{2y-1}$

(b) $\frac{\cos x}{1-2y}$

(c) $\frac{\sin x}{1-2y}$

(d) $\frac{\sin x}{2y-1}$

Q.6. Integrating factor of the differential equation $(\cot x)y'+y = \operatorname{cosec} x$

(a) $\cos x$

(b) $\tan x$

(c) $\sec x$

(d) $\sin x$

Q.7. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px+qy$, where $p, q > 0$ Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(a) $p = 2q$

(b) $p = \frac{q}{2}$

(c) $p = 3q$

(d) $p = q$

Q.8. The vectors $\lambda\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}-\hat{j}+\lambda\hat{k}$ are perpendicular if :

(a) $\lambda = 4$

(b) $\lambda = \frac{1}{4}$

(c) $\lambda = -\frac{1}{4}$

(d) $\lambda = -4$

Q.9. The direction cosines of $-2\hat{i}+\hat{j}-5\hat{k}$ are

(a) $< \frac{-2}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{-5}{\sqrt{10}} >$

(b) $< \frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}} >$

(c) $< -\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, -\frac{5}{\sqrt{30}} >$

(d) $< \frac{2}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{5}{\sqrt{10}} >$

Q.10. $\int \frac{x^3}{x+1} dx$ is equal to

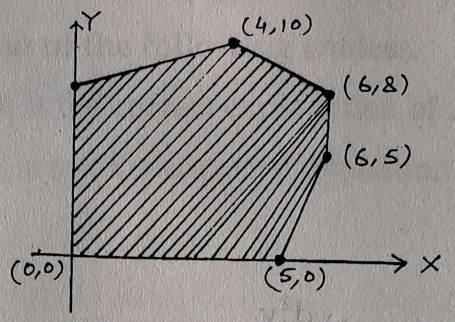
(a) $x+\frac{x^2}{2}+\frac{x^3}{3}-\ell og|1-x|+c$

(b) $x+\frac{x^2}{2}-\frac{x^3}{3}-\ell og|1-x|+c$

(c) $x-\frac{x^2}{2}-\frac{x^3}{3}-\ell og|1+x|+c$

(d) $x-\frac{x^2}{2}+\frac{x^3}{3}-\ell og|1+x|+c$

Q.11. The feasible solution for LPP is shown in following figure. Let $Z = 3x-4y$ be the objective function. Minimum of Z occurs at



(a) (0, 0)

(b) (0, 8)

(c) (5, 0)

(d) (4, 10)

Q.12. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x + y is :

(a) 2

(b) 5

(c) 6

(d) 4

Q.13. If A is a square matrix of order 3 and $|A| = 5$, then $|adjA| =$

(a) 5

(b) 25

(c) 125

(d) $1/5$

Q.14. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equal to :

(a) $\frac{4}{15}$

(b) $\frac{8}{45}$

(c) $\frac{1}{3}$

(d) $\frac{2}{9}$

Q.15. The general solution of differential equation ydx - xdy = 0

(a) $xy = c$

(b) $x = cy^2$

(c) $y = cx$

(d) $y = cx^2$

Q.16. If $y = \sin^{-1}x$ then $(1-x^2)\frac{d^2y}{dx^2}$ is equal to:

(a) $x\frac{dy}{dx}$

(b) xy

(c) $x\frac{d^2y}{dx^2}$

(d) x^2

Q.17. The number of vectors of unit length perpendicular to the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is :

(a) one

(b) two

(c) three

(d) infinite

Q.18. If the direction cosine of a line are k, k, k then :

(a) $k > 0$

(b) $0 < k < 1$

(c) $k = 1$

(d) $k = \frac{1}{\sqrt{3}}$ or $k = \frac{-1}{\sqrt{3}}$

ASSERTION - REASON BASED QUESTION

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q.19. **Assertion (A) :** The function given by $f(x) = \cos x$ is decreasing in $(0, \pi)$.

Reason (R) : A function f is decreasing in the internal [a, b] if $f'(x) < 0 \ \forall x \in (a, b)$

Q.20. **Assertion (A) :** The acute angle between the lines $\bar{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\bar{r} = 3\hat{k} + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$ is $\cos^{-1}\left(\frac{4}{9}\right)$

Reason (R) : The acute angle θ between the lines $\bar{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(a_1\hat{i} + b_1\hat{j} + c_1\hat{k})$ and $\bar{r} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} + \mu(a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

SECTION- B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

Q.21. Find the value of $\tan^{-1}(-1)$ in the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

OR

Show that the function $f : N \rightarrow N$ given by $f(x) = 2x$ is one-one but not onto.

Q.22. The volume of a sphere is increasing at the rate of 3 cm^3/sec. find the rate of increase of its surface area, when the radius is 2 cm.

Q.23. Show that the vector $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 4\hat{k}$ form right angled triangle.

OR

Find the direction ratio and direction cosine of the line passing through two points (2, - 4, 5) and (0, 1, -1).

Q.24. If $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\cdot\sqrt{1-x^2}\right]$, and $0<x<1$ then find $\frac{dy}{dx}$

Q.25. Find the values of x for which the angle between the vectors $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $\vec{b}=7\hat{i}-2\hat{j}+x\hat{k}$ is obtuse.

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q.26. Using properties of definite integration evaluate : $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

Q.27. Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

OR

A bag contains 5 red and 4 black balls and second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn together atrandom, both of which are found to be red. Find the probability that the balls are drawn from the second bag.

Q.28. Evaluate $\int_1^2\frac{\log x}{x^2}dx$

OR

Find $\int\sqrt{3-2x-2x^2}\ dx$

Q.29. Find the particular solution of the differential equation $\frac{dy}{dx}+\operatorname{cosec}\left(\frac{y}{x}\right)-\frac{y}{x}=0$, $y=0$ when $x=1$.

OR

Solve the differential equation

$(1+x)\,(1+y^2)\,dx+(1+y)\,(1+x^2)\,dy=0$

Q.30. Solve the following problem graphically

minimize $Z = 5x + 10y$

subject of constraints

$x + 2y \le 120,\ x + y \ge 60,\ x - 2y \ge 0$ and $x, y \ge 0$

A factory makes an open cardboard box for a shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps. If x is the length of each square cut from corners then :

(i) Find the volume of open box in terms of x.

(ii) What should be the side of square to be cut off so that volume is maximum ?

(iii) Find the maximum volume of open box.

OR

(iii) Find the total area of the removed squares.

Q.37. A laptop manufacturing company manufactures laptops at two plants located at different locations. The laptops manufactured are of two types, type A and type B. Production of both types at both locations are as below :

	Type A	**Type B**
Plant I	5600	1400
Plant II	2700	300

A laptop is randomly chosen and is found to be of type A. Based on the above information, answer the following :

(i) What is the probability that selected laptop is produced at plant-I ?

(ii) What is the probability that selected laptop is produced at plant-II ?

(iii) What is the probability that selected laptop is of type A ?

OR

(iii) What is the probability that selected laptop is of type B ?

Q.38. For a function f(x), if $f(-x) = f(x)$ then f(x) is even function and $f(-x) = -f(x)$ then f(x) is odd function . Again $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd} \end{cases}$

On the basis of above information answer the following questions :

i) If $f(x) = x^2 \sin x$ then check whether f(x) is even or odd function.

ii) Evaluate : $\int_{-\pi/2}^{\pi/2} |\sin x| dx$

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Q.31. Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

SECTION-D

(This section comprises of long answer type questions (LA) of 5 marks each)

Q.32. Sketch the region $\{(x, y): 0 \le 2y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$. Find the area of region using integration.

Q.33. Let A={1, 2, 3 ..... 9} and R be the relation in A×A defined by (a, b) R (c, d).
If a+d = b+c
for (a, b), (c, d) in A×A, prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)]

OR

If N denotes set of all natural numbers and R is the relation on N×N defined by (a, b) R(c, d) if ad(b+c) = bc(a+d). Show that R is an equivalence relation.

Q.34. Find the distance between the lines ℓ_1 and ℓ_2 given by

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$

OR

Find the foot of perpendicular from P(1, 2, –3) to the line $\frac{x+1}{2} = \frac{y+3}{-2} = \frac{z}{-1}$.

Also find the image of P in the given line.

Q.35. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & 20 \end{bmatrix}$, find A^{-1} using A^{-1}, solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$ and $\frac{6}{x} + \frac{9}{y} + \frac{20}{z} = -4$

SECTION - E

(This section comprises of 3 case-study/passage based & question of 4 marks each with two sub parts. First two case study question have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study questions has two sub parts of a marks each)

Q.36. **Case-study 1 :** Read the following passage and answer the questions given below :